

1. What is $Im(z)$, the imaginary part of z , if $z - \bar{z} = 10i$?
(A) 5 (B) -5 (C) 10 (D) -10 (E) NOTA
2. Two complex numbers $z_1 = 5 + 7i$ and $z_2 = -1 + 4i$ are plotted on the complex plane. Find the complex number that divides the line segment $\overline{z_1 z_2}$ by 1:2 ratio with shorter segment nearer z_1 .
(A) $2 + \frac{11}{2}i$ (B) $2 + 5i$ (C) $3 + 6i$ (D) $1 + 6i$ (E) NOTA
3. Let z, w be two complex numbers with $|z| = 2$ and $|w - 6 + 8i| = 5$. What is the smallest possible value of $|z - w|$?
(A) 3 (B) 5 (C) 10 (D) 17 (E) NOTA
4. Let z be a complex number with $|z| = 10$. Which of the following is equal to $\frac{z}{25}$?
(A) $\frac{4}{z}$ (B) $\frac{\bar{z}}{4}$ (C) $4z$ (D) $\frac{1}{4\bar{z}}$ (E) NOTA
5. If $z = 1 - i$ and $w = \sqrt{3} + i$, what is the argument of $\frac{w}{z}$?
(A) $\frac{\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $\frac{7\pi}{12}$ (D) $\frac{11\pi}{12}$ (E) NOTA
6. Simplify: $(-1 + i)^{10}$
(A) 32 (B) -32 (C) $32i$ (D) $-32i$ (E) NOTA
7. Let $z = a + bi$ be the complex number obtained by rotating $2 + 4i$ by 135° counterclockwise about the origin. What is ab ?
(A) 6 (B) -6 (C) 4 (D) -4 (E) NOTA
8. Simplify: $\frac{10i}{(1-i)(2-i)(3-i)}$
(A) $-i$ (B) i (C) -1 (D) 1 (E) NOTA
9. What is the area of the region enclosed by a closed curve z in the complex plane if $|z - \sqrt{3} - i\sqrt{2}| = 13$?
(A) 13π (B) 100π (C) 144π (D) 169π (E) NOTA
10. Find $a + b$ if two real numbers a and b satisfy $a(1 + 2i) + b(2 - i) = 8 + 6i$.
(A) 6 (B) 8 (C) 12 (D) 14 (E) NOTA
11. Let z be a complex root of $z^5 - 1 = 0$. Which one of the following is equal to $1 + z + z^2 + \dots + z^{2018} + z^{2019}$?
(A) i (B) $-i$ (C) 1 (D) 0 (E) NOTA

12. Let z and w be two nonzero complex numbers satisfying $z + \bar{z} = 0$ and $w + \bar{w} = 0$. What is the largest possible positive argument of $\frac{z}{w}$ less than 2π ?
- (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$ (E) NOTA
13. For a complex number z , if the real part of $\frac{z-1-i}{z+1+i}$ is 0, what is the distance from the origin to the point z in the complex plane?
- (A) $\sqrt{2}$ (B) $\frac{\pi}{2}$ (C) π (D) $\frac{3\pi}{2}$ (E) NOTA
14. Consider the equation $z^6 + z^4 - z^3 + z^2 + 1 = 0$. Which of the following statement(s) is true?
- a) $z^6 + z^4 - z^3 + z^2 + 1$ has three distinct integer factors of order 2.
b) There are exactly 6 distinct roots over complex number system, which are three pairs of complex conjugates.
c) The sum of the imaginary parts of all roots is positive.
- (A) b (B) b and c (C) a and b (D) a (E) NOTA
15. Given three vertices $4 + i, -1 - 2i, 2 + 7i$ of a parallelogram, which one of the following complex numbers can be the fourth vertex?
- (A) $1 + i$ (B) $7 + 10i$ (C) $-4 - 5i$ (D) $-5 - 4i$ (E) NOTA
16. Let m and n be the smallest positive integers such that $(1 + i\sqrt{3})^m = (1 - i)^n$. What is the value of $m + n$?
- (A) 12 (B) 24 (C) 36 (D) 48 (E) NOTA
17. If $2 + i$ is a root of $f(x) = x^3 + ax^2 + bx - 20$ where a and b are real numbers, what is the value of $a + b$?
- (A) -5 (B) 5 (C) -13 (D) 13 (E) NOTA
18. Let z_1 and z_2 be two solutions of the quadratic equation $x^2 - 2x + 2 = 0$. If z is a complex number such that $\Delta z z_1 z_2$ forms an equilateral triangle, what is the sum of all possible values of z ?
- (A) 2 (B) 0 (C) $2\sqrt{3}$ (D) $\sqrt{3}$ (E) NOTA
19. Let z_1, z_2, z_3 be three complex numbers with $|z_1 - z_2| = 7$ and $|z_2 - z_3| = 4$. If we let M and m be the maximum distance and the minimum distance between z_1 and z_3 , respectively, what is $M + m$?
- (A) 11 (B) 12 (C) 13 (D) 14 (E) NOTA
20. Let z_1, z_2, z_3, z_4, z_5 be 5 vertices on the unit circle that form a regular pentagon. What is the product of the distances from one vertex to each of the other 4 vertices?
- (A) 4 (B) 6 (C) 8 (D) 10 (E) NOTA

21. For how many real numbers x is $(x + i)^4$ real?
(A) 1 (B) 2 (C) 3 (D) 4 (E) NOTA
22. Let $w = \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}$. Which one of the following is NOT true?
(A) $w^2 = \bar{w}$ (B) $w^3 = -1$ (C) $\bar{w} = 1/w$ (D) $w^2 = -w - 1$ (E) NOTA
23. Which one of following best describes the graph of the equation $|z| + |z - 2 + 4i| = 3$ in the complex plane?
(A) A line (B) A circle (C) An ellipse (D) A parabola (E) NOTA
24. Let z be a complex number and \bar{z} be the complex conjugate of z . If both $\frac{z}{10}$ and $\frac{10}{\bar{z}}$ have real and imaginary parts between 0 and 1, inclusive, what is the smallest value of $|z|$?
(A) $\sqrt{2}$ (B) $5\sqrt{2}$ (C) 10 (D) 25 (E) NOTA
25. When $i - \frac{1}{i}$ is a root of a quadratic equation with real coefficients, what is the other root of the same equation?
(A) $i + \frac{1}{i}$ (B) $2i$ (C) $-\frac{2}{i}$ (D) $\frac{2}{i}$ (E) NOTA
26. If $f(n) = \left(\frac{1+i}{1-i}\right)^n + \left(\frac{1-i}{1+i}\right)^n$, find the sum $\sum_{n=1}^{2018} f(n)$.
(A) 2 (B) -2 (C) $2i$ (D) $-2i$ (E) NOTA
27. Assume that z_1, z_2, z_3 are complex numbers with $\frac{z_2 - z_1}{z_3 - z_1} = \sqrt{3} + i$. If the area of the triangle $\Delta z_1 z_2 z_3$ is equal to 18, what is $|z_3 - z_1|$?
(A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA
28. Let z and w be two nonzero complex numbers satisfying $z^6 + z^3 + 1 = 0$ and $w^6 - w^3 + 1 = 0$. How many distinct complex numbers are possible for the value of zw ?
(A) 6 (B) 9 (C) 12 (D) 18 (E) NOTA
29. Let z_1 be the root of $z^5 = 1$ with the smallest positive imaginary part. Let z_2 be the root of $z^7 = 1$ with the smallest positive imaginary part. What is the argument of $z_1 z_2$?
(A) $\frac{2\pi}{35}$ (B) $\frac{12\pi}{35}$ (C) $\frac{24\pi}{35}$ (D) $\frac{58\pi}{35}$ (E) NOTA
30. Let x and y be two nonzero complex numbers satisfying $x^2 + xy + y^2 = 0$. What is the value of $\left(\frac{x}{x+y}\right)^{100} + \left(\frac{y}{x+y}\right)^{100}$?
(A) 0 (B) -1 (C) 1 (D) 2 (E) NOTA